

What mass are the smallest protohalos?

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We calculate the kinetic-decoupling temperature for weakly interacting massive particles (WIMPs) in supersymmetric (SUSY) and universal-extra-dimension (UED) models that can account for the cold-dark-matter abundance determined from cosmic microwave background measurements. Depending on the parameters of the particle-physics model, a wide variety of decoupling temperatures is possible, ranging from several MeV to a few GeV. These decoupling temperatures imply a range of masses for the smallest protohalos much larger than previously thought — ranging from $10^{-6} M_{\oplus}$ to $10^2 M_{\oplus}$. We expect the range of protohalos masses derived here to be characteristic of most particle-physics models that can thermally accommodate the required relic abundance of WIMP dark matter, even beyond SUSY and UED.

PACS numbers: 12.60, 13.15, 98.80, 98.65

The physical nature of dark matter remains one of the major unsolved problems in theoretical physics and cosmology. One of the leading candidates for dark matter is a weakly interacting massive particle (WIMP) [1]. In the simplest models, WIMPs (which we denote by X) carry a conserved quantum number that renders them stable. When the temperature in the early Universe drops below m_X , the WIMP abundance creeps down the Boltzmann tail until pairs of WIMPs can no longer find each other within a Hubble time, and the comoving number density of WIMPs becomes constant. Up to factors of a few, this freeze-out of the annihilation channel happens at a temperature $T_{\text{fo}} \sim m_X/20$ and leads to a relic abundance of dark matter of $\Omega_X h^2 \simeq (3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}) / \langle \sigma_a v \rangle$, where $\langle \sigma_a v \rangle$ is the thermally averaged cross section (times relative velocity) for annihilation of X pairs into lighter particles. In typical models, $m_X \sim 100 - 1000 \text{ GeV}$, and $T_{\text{fo}} \sim 5 - 50 \text{ GeV}$.

While freeze-out signals the departure of WIMPs from *chemical* equilibrium, it does *not* signal the end of WIMP interactions. Elastic and inelastic scattering processes of the form $Xf \rightarrow Xf$ or $Xf \rightarrow X'f'$ keep the dark matter in *kinetic* equilibrium until later times (lower temperatures) [2, 3, 4]. Here f and f' are SM particles in the thermal bath (leptons, quarks, gauge bosons) and X' is an unstable particle that carries the same conserved quantum number as X . The temperature T_{kd} of *kinetic* decoupling sets the distance scale at which linear density perturbations in the dark-matter distribution get washed out—the small-scale cutoff in the matter power spectrum. In turn, this small-scale cutoff sets the mass M_c of the smallest protohalos that form when these very small-scales go nonlinear at a redshift $z \sim 70$. There may be implications of this small-scale cutoff for direct [5] and indirect [6] detection.

Some early work assumed that the cross sections for WIMPs to scatter from light particles (e.g., photons and neutrinos) would be energy independent, leading to suppression of power out to fairly large (e.g., galactic)

scales. However, in supersymmetric models, at least, the relevant elastic-scattering cross sections drop precipitously with temperature, resulting in much higher T_{kd} and much smaller suppression scales [3]. If the annihilation cross section of WIMPs into light fermions goes as $\sigma_a \simeq g_a^4/m_X^2$, then one expects the scattering cross section to be $\sigma_s \simeq g_s^4 E^2/m_X^4$, where E is the energy of the scattering light particles, and $g_a \sim g_s$ up to factors of order unity. This estimate has been used to derive T_{kd} and infer that the minimum protohalo mass is $M_c \sim M_{\oplus}$ [4, 5, 7]. However, to date, no detailed calculation of T_{kd} and M_c in supersymmetric or other models consistent with experimental and cosmological data have been performed.

In this *Letter*, we calculate the kinetic-decoupling temperature T_{kd} of WIMP dark matter in models that account for the correct cold-dark-matter density while remaining consistent with laboratory constraints. We consider models within the minimal supersymmetric extension of the standard model (MSSM) and models with universal extra dimensions (UED). Instead of relying on heuristic arguments or toy models, we use the detailed scattering cross sections of WIMPs, including resonances and threshold effects, both for the WIMP relic abundances and for T_{kd} . The main result of our analysis is that T_{kd} may range all the way from tens of MeV to several GeV. These T_{kd} imply a range $M_c \sim 10^{-6} M_{\oplus}$ to $M_c \sim 10^2 M_{\oplus}$, where we use the estimate [7]

$$M_c \simeq 33.3 (T_{\text{kd}}/10 \text{ MeV})^{-3} M_{\oplus}, \quad (1)$$

which accounts for both the acoustic oscillations imprinted on the power spectrum by the coupling between the dark matter and the relativistic particles in the primordial plasma prior to kinetic decoupling and the cutoff due to free-streaming of dark matter after kinetic decoupling. Although we focus on particular WIMP scenarios, we expect the range of M_c derived here will be characteristic of most particle-physics models that can accommodate the required relic abundance of thermal WIMP

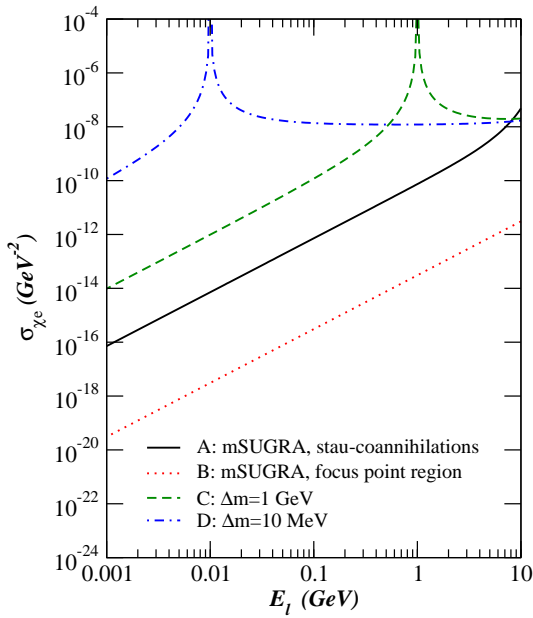


FIG. 1: Neutralino-electron scattering cross section as a function of the electron energy E_l for the four benchmark models **A-D** discussed in the text.

dark matter.

We define T_{kd} from $\tau_r(T_{\text{kd}}) = H^{-1}(T_{\text{kd}})$ [4], where $H(T)$ is the Hubble expansion rate, and the relaxation time τ_r is

$$\tau_r^{-1} \equiv \sum_l n_l(T, m_l) \sigma_{lX}(T) (T/m_X). \quad (2)$$

Here, $n_l(T, m_l) \sim T^3$ is the equilibrium number density of the relativistic particle species l (the true mass dependence can be crucial here for some of the species under consideration such as the μ and τ leptons), $\sigma_{lX}(T)$ is the thermally averaged scattering cross section of the WIMP X off l 's, and the factor $(m_X/T)^{-1}$ counts the number of scatters needed to keep the WIMPs in kinetic equilibrium. Here, we consider $l \in \{\nu_{e,\mu,\tau}, e^\pm, \mu^\pm, \tau^\pm\}$ and neglect the scattering off light quarks. This is well justified for temperatures $T_{\text{kd}} \ll m_\pi < \Lambda_{QCD}$, as the scattering of WIMPs off mesons and hadrons is suppressed with respect to their scattering off light leptons by the relative abundance of the species in the thermal bath. In fact, in some cases, we find $T_{\text{kd}} \gtrsim m_\pi$, and strictly speaking in these cases a detailed model for the confinement mechanism should be included. Here, we neglect these effects and give what can be regarded, in this regime, as an upper limit to T_{kd} (taking into account the scattering off strongly-interacting particles would in fact *decrease* T_{kd}).

In the case of supersymmetric models, the scattering of neutralinos (χ 's) off leptons proceeds through sfermion and gauge-boson exchange. The relevant cross sections have been computed in Ref. [3] for the case of neutralino-

neutrino scattering. We extend here the results of Ref. [3] to include charged-lepton scattering, where further diagrams (involving both right- and left-handed charged-slepton exchange) as well as novel interfering amplitudes appear. The scattering cross section $\sigma_{\chi l}$ goes as E_l^2 [3], modulo resonant channels where the exchanged slepton mass is quasi-degenerate with the neutralino mass. In this latter case, the slepton width has to properly be taken into account in the computation of $\sigma_{\chi l}(T)$.

We show our results for the neutralino-electron scattering cross section as a function of energy in Fig. 1, where we pick supersymmetric “benchmark” models in the context of the minimal supergravity (mSUGRA) paradigm [8]. We set for all models (with the usual notation) $m_{1/2} = 500$ GeV, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$, and $m_t = 172.7$ GeV; in all cases, the neutralino mass is around 200 GeV. Model **A** features $m_0 = 100$ GeV, and lies in the *coannihilation region*, where scalar superparticles are light, and the next-to-lightest supersymmetric particle (NLSP) is a τ slepton. The latter is, here, quasi-degenerate with the lightest neutralino, and coannihilation among the two species brings the neutralino relic abundance into accord with the dark-matter abundance. Model **B** belongs, instead, to the *focus-point region*, where large scalar masses (here, $m_0 = 2770$ GeV) at the grand-unification scale drive the higgsino mass parameter μ to low values at the weak scale through renormalization-group evolution and radiative electroweak-symmetry breaking. A low value of μ implies a mixed higgsino-bino dark-matter particle, which again can produce a thermal relic abundance in the cosmological density range. Heavy sfermions imply that scattering off light fermions proceeds through Z^0 exchange, and the resulting $\sigma_{\chi l}$ is suppressed with respect to the light-sfermion case (model **A**) by almost four orders of magnitude.

We also examine models that exhibit the effects of sfermion resonances in neutralino-lepton scattering. We modify model **A**, lowering the soft-supersymmetry-breaking left-handed slepton masses of the first two generations, in order to get $\Delta m_{\tilde{\nu}_{e,\mu}} \equiv m_{\tilde{\nu}_{e,\mu}} - m_\chi \simeq \Delta m_{\tilde{e}_1, \tilde{\mu}_1} = 1$ GeV (model **C**) and 0.01 GeV (model **D**). These models can be motivated in the context of extensions of mSUGRA with non-universal scalar masses (see, e.g., Ref. [9]). At sufficiently small temperatures, $\sigma_{\chi l} \propto E_l^2$ is recovered, but for $T \gtrsim \Delta m_{\tilde{\nu}}$, $\sigma_{\chi l} \simeq \text{constant}$, and is simply set by the neutralino mass and by the relevant neutralino-lepton-slepton couplings ($\sigma_{\chi l} \propto |g_{\chi \tilde{l} l}|^4/m_\chi^2$).

Another class of WIMP models that has recently received considerable attention is that arising in the context of universal extra dimensions (UED) [10]. In minimal setups, UED features a stable lightest Kaluza-Klein particle (LKP) whose nature is model dependent. Candidate LKPs include the first Kaluza-Klein (KK) excitations of the $U(1)$ gauge boson and the neutrino ($B^{(1)}$ and $\nu^{(1)}$ respectively). Precision electroweak measure-

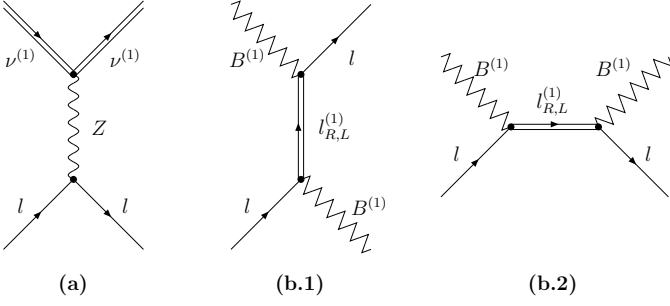


FIG. 2: Feynman diagrams contributing to the scattering of $\nu^{(1)}$ (a) and $B^{(1)}$ (b.1 and b.2) off leptons.

ments [11], the LKP relic abundance [12], and direct-detection experiments [13] strongly constrain the viable ranges of masses for LKPs. However, the allowed range of masses for the LKP sensitively depends upon the details of the spectrum of the first and second KK excitations, which can include significant coannihilation and resonant-annihilation effects. We compute here the scattering cross section of $\nu^{(1)}$ and of $B^{(1)}$ off leptons, for which the relevant Feynman diagrams are shown in Fig. 2. In the case of the $B^{(1)}$, we expect large scattering cross sections, since the intrinsically degenerate nature of the KK spectrum, where $m_{B^{(1)}} \simeq m_{L^{(1)}}$, clearly enforces a resonant enhancement. We find, to leading order in E_l/m_X , and in the relativistic limit for l and non-relativistic limit for the LKP particle, that

$$\sigma_{\nu^{(1)}l} \simeq \frac{|g_{\nu^{(1)}\nu^{(1)}Z}|^2}{4\pi m_Z^4} (g_L^2 + g_R^2) E_l^2, \quad (3)$$

$$\sigma_{B^{(1)}l} \simeq \frac{E_l^2}{2\pi} \sum_{R,L} \frac{(g_1 Y_{R,L})^4}{(m_{B^{(1)}}^2 - m_{l_{R,L}^{(1)}}^2)^2}, \quad (4)$$

where $g_{R,L}$ stand for the L and R couplings of the lepton l to the Z^0 gauge boson, $Y_{R,L}$ for the hypercharge quantum number, and $g_{\nu^{(1)}\nu^{(1)}Z} = e/(\sin 2\theta_W)$. The $\sigma_{Xl} \propto E_l^2$ scaling found in the case of neutralino dark matter is valid for this alternative class of WIMPs as well. In the case of the KK neutrino, further, $\sigma_{\nu^{(1)}l}$ does not depend on the LKP mass. We stress that consistency with direct-detection experiments requires $m_{\nu^{(1)}} \gtrsim 50$ TeV [13]. While this latter range is in conflict with estimates of the thermal relic abundance of $\nu^{(1)}$ [12], the particle properties of the latter, assuming the coupling $g_{\nu^{(1)}\nu^{(1)}Z}$ with the Z^0 , to be a free parameter instead of being fixed by the standard gauge interactions, apply to other dark-matter candidates including the Dirac right-handed neutrino of 5D warped grand unification [14].

Different WIMP models give rise to different T_{kd} , and therefore to different M_c . We apply our elastic-scattering cross section for WIMPs from light leptons to the MSSM parameter space, following the scan procedure of Ref. [15], requiring that the neutralino density

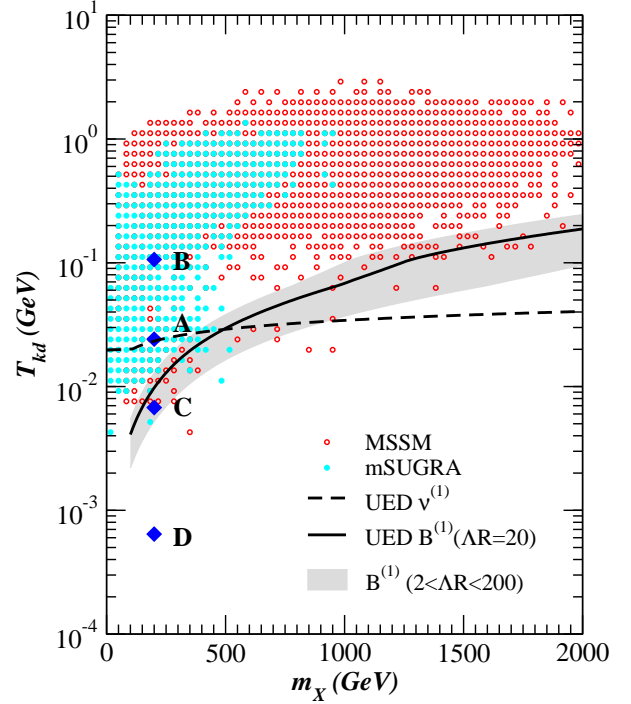


FIG. 3: The kinetic-decoupling temperature T_{kd} as a function of the WIMP mass for supersymmetric models (red empty dots are for the general MSSM while light-blue filled dots are for mSUGRA) giving a neutralino thermal relic abundance consistent with cosmology, and for UED models featuring a $B^{(1)}$ and a $\nu^{(1)}$ LKP. The four benchmark models **A-D** discussed in the text are also shown.

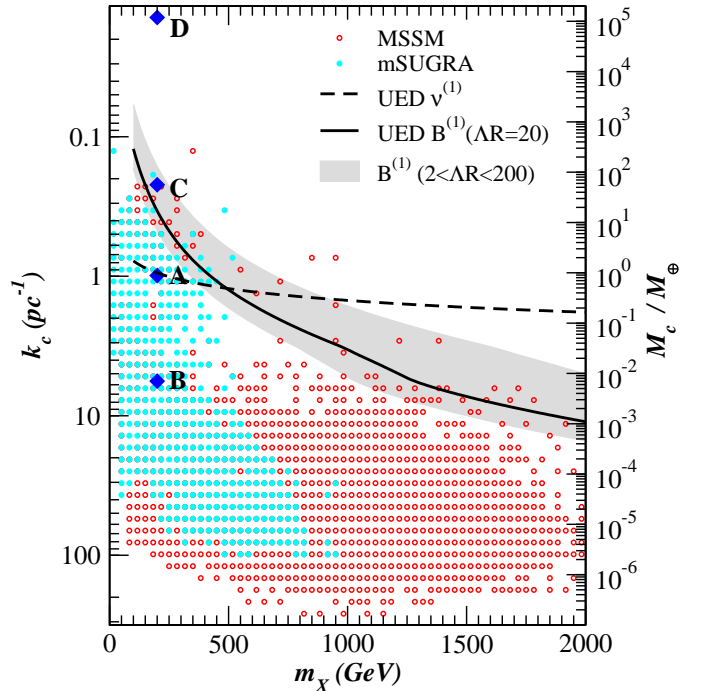


FIG. 4: The WIMP protohalo characteristic comoving wavenumber k_c (left axis) and mass M_c (right axis) as a function of the WIMP mass, for the same models as in Fig. 3.

falls within the WMAP 2σ range for the dark-matter density [16]. Figs. 3 and 4 show our results for T_{kd} and for M_c , respectively, versus the WIMP mass. We also scan over a subset of the MSSM, defined by the mSUGRA setup: the results of this second scan are shown as light blue filled dots in the figures. Finally, we indicate the range of results expected for a $B^{(1)}$ and a $\nu^{(1)}$ LKP. We set the KK spectrum according to the minimal UED prescription for radiative corrections to the KK masses [17], setting the cutoff scale $2 < \Lambda R < 200$, and showing the “standard” $\Lambda R = 20$ case with a black solid line. We also indicate the four benchmark models of Fig. 1 discussed above. We conclude that (1) in most SUSY models, $10 \lesssim T_{\text{kd}}/\text{MeV} \lesssim 4000$, but even lower values can be attained with finely-tuned resonant channels; (2) the expected range of M_c varies over the wide range $10^{-6} \lesssim M_c/M_\oplus \lesssim 10^2$; (3) $B^{(1)}$ LKPs typically decouple later than neutralinos (around 10 MeV for values of $m_{B^{(1)}}$ preferred by the thermal relic abundance and by electroweak measurements); and (4) X particles scattering off light leptons through a Z^0 exchange with a coupling equal to g_{XXZ} produce protohalos with mass

$$M_c \approx 100 M_\oplus |g_{XXZ}|^{3/2} (m_X/100 \text{ GeV})^{-3/4}. \quad (5)$$

Since the relevant quantities for WIMP-nucleon scattering and for the annihilation of WIMPs into gamma rays or antimatter are typically poorly correlated with WIMP-lepton scattering, conventional dark-matter-detection rates cannot be simply related to T_{kd} . However, we point out that low T_{kd} imply large WIMP-lepton scattering cross sections and may produce sizable signals at future electron accelerators using the search technique recently proposed in Ref. [18]. If the masses of the sleptons and neutralino can be measured by future colliders then, if Δm is small enough, the beam energy of a future electron accelerator might be tuned to $E_{\text{beam}} \simeq \Delta m$, enabling resonant s -channel scattering of neutralino dark-matter. The scattered electrons could then be detected by calorimeters or tracking chambers along the beam line. This technique would also reveal information about the dark-matter velocity distribution [18]. As a rule of thumb, more than one event per year is expected at a future electron collider with 100 m of detector length and 10 A of beam current if $T_{\text{kd}} \lesssim 10$ MeV; in the extreme case of $T_{\text{kd}} \approx 1$ MeV, the expected event rate per year could be as large as $\sim 10^4$!

The temperature T_{kd} in WIMP models has a critical impact not only on the size distribution of primordial protohalos expected in N -body simulations of structure formation [5], but also for potential effects in WIMP direct and indirect detection induced by dark matter clumps or streams, or in the anisotropy of the cosmic gamma-ray background induced by WIMP annihilations [6]. Accounting for the wide range of possibilities consistent with detailed WIMP models should therefore be

regarded as an essential ingredient for future studies in this field.

KS is supported by NASA through Hubble Fellowship grant HST-HF-01191.01-A awarded by the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA, under contract NAS 5-26555. SP and MK are supported in part by DoE DE-FG03-92-ER40701 and FG02-05ER41361 and NASA NNG05GF69G.

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- [1] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rept. **267**, 195 (1996) [arXiv:hep-ph/9506380]; L. Bergstrom, Rept. Prog. Phys. **63**, 793 (2000) [arXiv:hep-ph/0002126]; G. Bertone, D. Hooper, and J. Silk, Phys. Rept. **405**, 279 (2005) [arXiv:hep-ph/0404175].
- [2] C. Boehm, P. Fayet, and R. Schaeffer, Phys. Lett. B **518**, 8 (2001) [arXiv:astro-ph/0012504].
- [3] X. L. Chen, M. Kamionkowski, and X. M. Zhang, Phys. Rev. D **64**, 021302 (2001) [arXiv:astro-ph/0103452].
- [4] A. M. Green, S. Hofmann, and D. J. Schwarz, Mon. Not. Roy. Astron. Soc. **353**, L23 (2004) [arXiv:astro-ph/0309621]; A. M. Green, S. Hofmann, and D. J. Schwarz, JCAP **0508**, 003 (2005) [arXiv:astro-ph/0503387].
- [5] J. Diemand, B. Moore, and J. Stadel, Nature **433**, 389 (2005) [arXiv:astro-ph/0501589]; J. Diemand, M. Kuhlen and P. Madau, arXiv:astro-ph/0603250.
- [6] S. Ando and E. Komatsu, Phys. Rev. D **73**, 023521 (2006) [arXiv:astro-ph/0512217].
- [7] A. Loeb and M. Zaldarriaga, Phys. Rev. D **71**, 103520 (2005) [arXiv:astro-ph/0504112].
- [8] A. H. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. **49**, 970 (1982).
- [9] H. Baer *et al.*, JHEP **0406**, 044 (2004) [arXiv:hep-ph/0403214]; S. Profumo, Phys. Rev. D **68**, 015006 (2003) [arXiv:hep-ph/0304071].
- [10] T. Appelquist, H. C. Cheng, and B. A. Dobrescu, Phys. Rev. D **64**, 035002 (2001) [arXiv:hep-ph/0012100]; G. Servant and T. M. P. Tait, Nucl. Phys. B **650**, 391 (2003) [arXiv:hep-ph/0206071].
- [11] T. Flacke, D. Hooper, and J. March-Russell, arXiv:hep-ph/0509352.
- [12] K. Kong and K. T. Matchev, JHEP **0601**, 038 (2006) [arXiv:hep-ph/0509119]; F. Burnell and G. D. Kribs, Phys. Rev. D **73**, 015001 (2006) [arXiv:hep-ph/0509118]; M. Kakizaki *et al.*, Nucl. Phys. B **735**, 84 (2006) [arXiv:hep-ph/0508283].
- [13] G. Servant and T. M. P. Tait, New J. Phys. **4**, 99 (2002) [arXiv:hep-ph/0209262].
- [14] K. Agashe and G. Servant, JCAP **0502**, 002 (2005) [arXiv:hep-ph/0411254]; D. Hooper and G. Servant, Astropart. Phys. **24**, 231 (2005) [arXiv:hep-ph/0502247].
- [15] S. Profumo and C. E. Yaguna, Phys. Rev. D **70**, 095004 (2004) [arXiv:hep-ph/0407036].
- [16] D. N. Spergel *et al.* (WMAP Collaboration), Astrophys.

- J. Suppl. **148**, 175 (2003) [arXiv:astro-ph/0302209].
- [17] H. C. Cheng, K. T. Matchev, and M. Schmaltz, Phys. Rev. D **66**, 036005 (2002) [arXiv:hep-ph/0204342].
- [18] J. Hisano *et al.*, AIP Conf. Proc. **805**, 423 (2006) [arXiv:hep-ph/0504068].